Propagating Signals on Nerve Fibers

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The cells of the nervous system, the so–called *neurons*, are active units able to transmit ionic currents into or out of the cells through ion channels. The transport of ions is so profoundly regulated that it forms the basis for the propagation of nervous signals along the cell, yet the underlying physical principles are fairly simple. In this project we shall derive and study a mathematical model for the propagation of pulses in nerve fibers. The original work led to the 1963 Nobel Prize in Physiology to the authors Alan Hodgkin and Andrew Huxley.

Nerve fibres are amazing: They must transmit pulses without significant damping, yet with a speed sufficient to allow the agile movements of living creatures. Nerve signals are not the result of electron currents as in a coaxial cable, but a consequence of a wave phenomenon in the electrical potential across the cell membrane.

At rest, the neurons maintain a constant resting potential of about -65mV by replacing Na^+–ions from the interior of the cell with K^+–ions from the exterior. (This, interestingly, is also the basis for another Nobel Prize, the 1997 Chemistry prize, awarded (in part) to Jens Christian Skou and his work leading to the identification of the Na^+–K^+ pump.) Thus, the resting potential is a result of differences in the ionic concentrations in the neuron interior and exterior.

If, for some reason[1], the cell membrane is excited, an *action potential* is formed. This wave consists of appropriate ionic currents into and out of the cell axon with different time scales, and is the focus of this excercise.

Problem 1 *Read the references, at first [1, 2]. Explain how the physiological action potential propagates along the axon and how it is modelled.*

[1]Explaining this reason is probably worthy of another Nobel prize...

Problem 2 *Plot the initial gating variables n, m and h as well as the time constants τ_n, τ_m and τ_h as functions of the voltage, V.*

Problem 3 *Solve the Hodgkin–Huxley–system in the space–clamped case. Plot the conductances as function of time. Determine the action potential threshold (to an accuracy of 3 decimal digits) and refractory periods. Include temperature dependence.*

Problem 4 *Investigate analytically a system with a propagating pulse using Fischer's equation in the travelling wave formulation*

$$\frac{d^2u}{d\xi^2} + v\frac{du}{d\xi} = f(u) \tag{1}$$

with

$$f(u) = u(u-a)(u-1), \quad 0 < a < 1. \tag{2}$$

Consult [3] for inspiration and obtain an analytical expression for the kink–type solution of Fischer's equation (i.e. obtain [3, Eq. (6)]).

Problem 5 *Consider the full Hodgkin–Huxley–system for the propagating action potential, using the travelling wave approximation ([2], Eq. (31)) and simplify the resulting 5th order system to a second order system, using reasonable values for the gating variables n, m and h.*

Compare this second order system to the Fischer system of Problem 4, using an analogy with Fischer's $f(u)$ and Hodgkin–Huxley's current, $I(V)$, and thereby estimate the propagation velocity.

Problem 6 *Solve the full Hodgkin–Huxley–system numerically using a travelling wave assumption.*

If time permits, other questions to be investigated could be the FitzHugh–Nagumo system, effects of myelinisation or any related subject of interest.

References

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[2] A.L. Hodgkin and A.F. Huxley: *A quantitative description of membrane current and its application to conduction and excitation in nerve*, J. Physiol. **117**, pp. 500–544 (1952).

[3] A.C. Scott: *Nonlinear Science, Emergence & Dynamics of Coherent Structures*, Oxford University Press, 1999, pp. 118–123.

[4] J. Keener and J. Snyed: *Mathematical Physiology*, Springer, 1998, pp. 116–130.

[5] A. Despopoulos and S. Silbernagel: *Color Atlas of Physiology*, Thieme, 4th Ed., 1985, pp. 24–29.